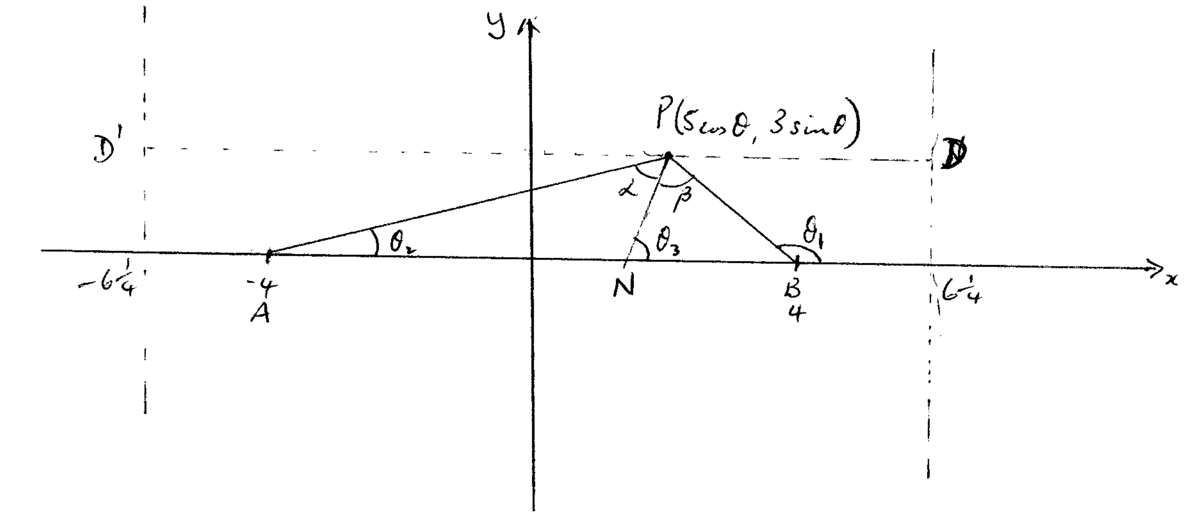
Q The point P lies on the ellipse with equation $9x^2 + 25y^2 = 225$, and A and B are points $(-4,0)$ and $(4,0)$ respectively.

a) Prove that $PA + PB = 10$

b) Prove also that the normal at P bisects the angle APB.



a)

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$\therefore \; 9 = 25(1 - e^2)$$

$$\therefore \; e = \frac{4}{5}$$

$$PB = ePD = \frac{4}{5}\left(\frac{25}{4} - 5\cos\theta\right) = 5 - 4\cos\theta$$

$$PA = ePD' = \frac{4}{5}\left(\frac{25}{4} + 5\cos\theta\right) = 5 + 4\cos\theta$$

$$\therefore \; AP + PB = 10$$

b) $\tan\theta_1 = \dfrac{3\sin\theta}{5\cos\theta - 4}$, $\tan\theta_2 = \dfrac{3\sin\theta}{5\cos\theta + 4}$, $\tan\theta_3 = \dfrac{5\sin\theta}{3\cos\theta}$.

$$\tan(\alpha + \beta) = \frac{\dfrac{3\sin\theta}{5\cos\theta - 4} - \dfrac{3\sin\theta}{5\cos\theta + 4}}{1 + \dfrac{9\sin^2\theta}{25\cos^2\theta - 16}} = \frac{24\sin\theta}{16\cos^2\theta - 7}$$

$$\tan\alpha = \frac{\dfrac{5\sin\theta}{3\cos\theta} - \dfrac{3\sin\theta}{5\cos\theta + 4}}{1 + \dfrac{15\sin^2\theta}{3\cos\theta(5\cos\theta+4)}}$$

$$= \frac{4\sin\theta}{3}$$

$$\therefore \tan 2\alpha = \frac{\dfrac{8\sin\theta}{3}}{1 - \dfrac{16\sin^2\theta}{9}}$$

$$= \frac{24\sin\theta}{16\cos^2\theta - 7}$$

$$\therefore 2\alpha = \alpha + \beta \Rightarrow \underline{\alpha = \beta}$$

$\therefore$ normal bisects angle APB