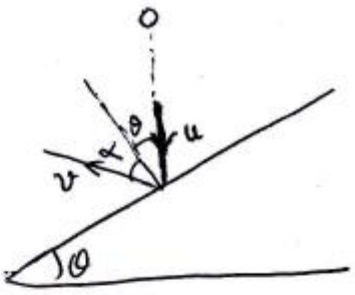
3 A ball of mass m kg is dropped from a vertical height of h m above a smooth plane that is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$.

The coefficient of restitution between the ball and the plane is e.

Given that the ball loses half of its kinetic energy on impact with the plane, find the value of e.

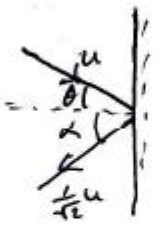
(9)



$\tan\theta = \frac{3}{4}$

KE before $= \frac{1}{2}mu^2$ KE after $= \frac{1}{2}mv^2$

$\therefore \frac{1}{2}mv^2 = \frac{1}{4}mu^2 \Rightarrow v = \frac{1}{\sqrt{2}}u$



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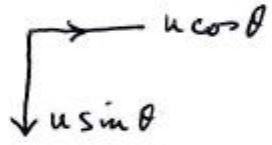

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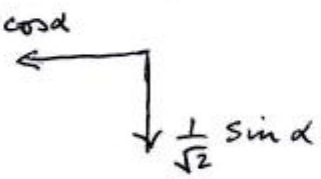


So: $\frac{1}{\sqrt{2}}u\sin\alpha = u\sin\theta \Rightarrow \frac{1}{\sqrt{2}}\sin\alpha = 0.6$

$\sin\alpha = \sqrt{2} \times 0.6$

$\alpha = 58.05^\circ$

$\therefore e = \frac{\frac{1}{\sqrt{2}}u\cos\alpha}{u\cos\theta}$

$\therefore e = \frac{0.528}{\sqrt{2} \times 0.8} = 0.4677$

7 A smooth sphere S is moving on a smooth horizontal plane with speed u when it collides with a smooth fixed vertical wall. At the instant of collision, the direction of motion of S makes an angle of $60°$ with the wall. Immediately after the collision S has a speed of $\frac{15}{16}u$ m s^{-1} and its direction of motion makes an angle of α with the wall.

a Find:

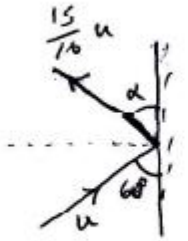
i the value of α

ii the coefficient of restitution between S and the wall. **(4)**

S then moves on and collides obliquely with another smooth sphere T of equal mass and radius. Immediately before the impact T is stationary and the velocity of S makes an angle of α with the lines of centres of the two spheres, where α is the angle found in part **a i**. The coefficient of restitution between the spheres is $\frac{3}{4}$.

b Find the speeds of S and T immediately after the collision in terms of u, and the angle the velocity of each sphere makes with the line of centres. **(5)**

a)



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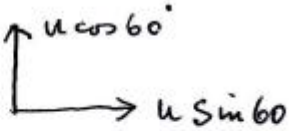

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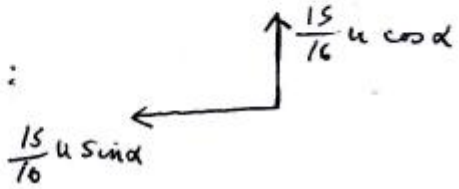
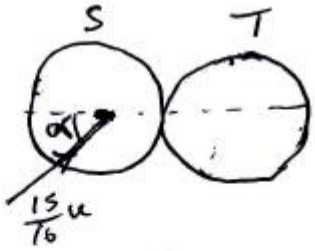


i) S_o : $\frac{15}{16}u\cos\alpha = \frac{1}{2}u$

$\cos\alpha = \frac{8}{15} \Rightarrow \alpha = 57.8°$

ii) $\therefore e = \dfrac{\frac{15}{16}\sin 57.8°}{\frac{\sqrt{3}}{2}} = 0.916$

b)



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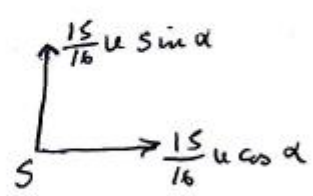

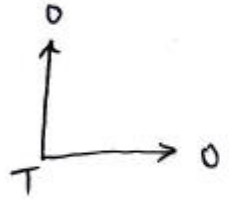

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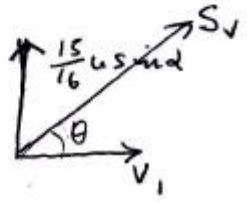

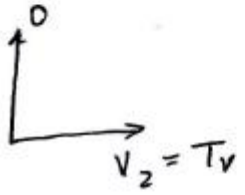

So: $\frac{15}{16} u \cos\alpha = V_1 + V_2$

$V_1 + V_2 = 0.5$

And: $e = \frac{V_2 - V_1}{\frac{15}{16} u \sin\alpha} = \frac{3}{4} \Rightarrow V_2 - V_1 = 0.375$

$\therefore 2V_2 = 0.875u \rightarrow V_2 = 0.4375u$

$V_1 = 0.0625u$

$\therefore T_V = 0.4375u$ along line of centres

$S_V = \sqrt{0.0625^2 + 0.7933^2}\, u = 0.7957$

$\theta = \tan^{-1}\left(\frac{0.7933}{0.0625}\right)$

$= 85.5°$ to line of centres.