$$I_n = \int_0^5 \frac{x^n}{\sqrt{25-x^2}}\,dx \qquad n \geq 0$$

a. find an expression for $\int \frac{x}{\sqrt{25-x^2}}\,dx \quad 0 \leq x \leq 5$

b. using your answer to part a or otherwise show that

$$I_n = \frac{25(n-1)}{n} I_{n-2} \qquad n \geq 2$$

c. find I_4 in the form of $k\pi$ where k is a fraction.

$$I_n = \int_0^5 \frac{x^n}{\sqrt{25-x^2}}\,dx$$

a) $$\int \frac{x}{\sqrt{25-x^2}}\,dx = -\frac{1}{2}\int \frac{-2x}{\sqrt{25-x^2}}\,dx = -\frac{1}{2}\int u^{-\frac{1}{2}}\,du = -\sqrt{25-x^2}$$

b) $$I_n = \int_0^5 \frac{x^n}{\sqrt{25-x^2}}\,dx = \int x^{n-1}\cdot\frac{x}{\sqrt{25-x^2}}\,dx$$

$$u = x^{n-1} \qquad dV = \frac{x}{\sqrt{25-x^2}}$$

$$du = (n-1)\,x^{n-2} \qquad V = -\sqrt{25-x^2}$$

$$I_n = \left[-x^{n-1}\sqrt{25-x^2}\right]_0^5 + \int (n-1)\,x^{n-2}\sqrt{25-x^2}\,dx$$

$$I_n = 0 + (n-1)\int x^{n-2}\cdot\frac{(25-x^2)}{\sqrt{25-x^2}}\,dx$$

$$I_n = (n-1)\left[25\int \frac{x^{n-2}}{\sqrt{25-x^2}}\,dx - \int \frac{x^n}{\sqrt{25-x^2}}\cdot dx\right]$$

$$I_n = (n-1)\left[25\,I_{n-2} - I_n\right]$$

$$\cancel{I_n} = 25(n-1)\,I_{n-2} - n\,I_n + \cancel{I_n}$$

$$n\,I_n = 25(n-1)\,I_{n-2}$$

$$I_n = \frac{25(n-1)}{n}\,I_{n-2} \qquad n \geq 2$$

c) $$I_0 = \int_0^5 \frac{1}{\sqrt{25-x^2}}\,dx = \left[\arcsin\left(\frac{x}{3}\right)\right]_0^5 = \frac{\pi}{2}$$

$$\therefore I_2 = \frac{25\,(2-1)}{2}\times I_0 = \frac{25}{4}\pi$$

$$\therefore I_4 = \frac{25\times 3}{4}\times I_2 = \frac{25\times 3}{4}\times\frac{25}{4}\pi = \frac{1875}{16}\pi$$