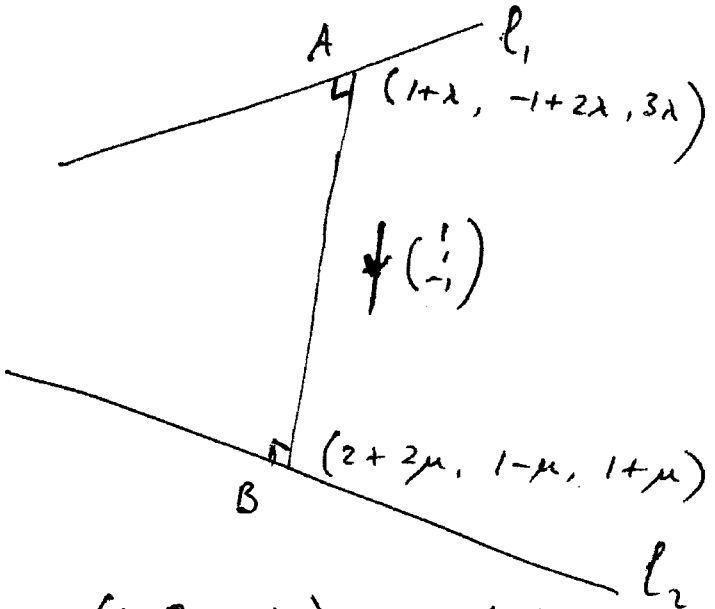
The line $l_1$ has equation $r = i - j + \lambda(i + 2j + 3k)$ and the line $l_2$ has equation $r = 2i + j + k + \mu(2i - j + k)$. The point A lies on $l_1$ and the point B lies on $l_2$. Given that AB is perpendicular to $l_1$ and $l_2$, find the coordinates of A and B.

$$l_1 : \underline{r} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$l_2 : \underline{r} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

Vector perpendicular to both lines: $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$




$$\overrightarrow{AB} = \begin{pmatrix} 1 + 2\mu - \lambda \\ 2 - \mu - 2\lambda \\ 1 + \mu - 3\lambda \end{pmatrix} = t \begin{pmatrix} 1 \\ +1 \\ -1 \end{pmatrix}$$

ie
$$1 + 2\mu - \lambda = t \quad (1)$$
$$2 - \mu - 2\lambda = t \quad (2)$$
$$1 + \mu - 3\lambda = -t \quad (3)$$

$2+3 \rightarrow \quad 3 - 5\lambda = 0 \Rightarrow \lambda = \frac{3}{5} \quad \therefore A = \left(1\frac{3}{5}, \frac{1}{5}, 1\frac{4}{5}\right)$

$1+3 \Rightarrow \quad 2 + 3\mu - 4\lambda = 0 \Rightarrow \mu = \frac{2}{15} \quad \therefore B = \left(2\frac{4}{15}, \frac{13}{15}, 1\frac{2}{15}\right)$